Diffeomorphism-Invariant Bulk Observables in AdS

Alex Kinsella, UCSB
Advised by Steve Giddings

Questions

1. How can we construct diffeomorphism-invariant bulk observables in asymptotically AdS spacetimes?

2. How does gravity give rise to holography?

Setup

- Consider quantum matter fields on a semiclassical AAdS background

$$ds^2_{d+1} = \frac{l^2}{\cos^2\rho}\left(-d\tau^2 + d\rho^2 + \sin^2\rho\, d\Omega^2_{d-1} + \kappa h_{\mu\nu} dx^\mu dx^\nu\right)$$

$\kappa^2 = 32\pi G$

- Extrapolate dictionary: Boundary observables are the rescaled boundary limits of diffeomorphism-invariant bulk operators

$$\mathcal{O}_{\text{bdry}} = \lim_{\rho\to\pi/2} (\cos\rho)^{-\Delta} \mathcal{O}_{\text{bulk}}$$

- Question: How is algebra of boundary observables related to algebra of bulk observables?

Gravitational Dressing

- Consider a bulk scalar field $\phi(x)$. It is not diffeomorphism invariant on its own.

	Diff	$\longrightarrow$	Pushforward
Globally:	$f : M \to M$		$\phi(x) \to \phi(f^{-1}(x))$
Infinitesimally:	$\kappa\xi^\mu$		$\phi(x) \to \phi(x) - \kappa\xi^\mu\partial_\mu\phi + \mathcal{O}(\kappa^2)$

- Solve by gravitationally dressing:

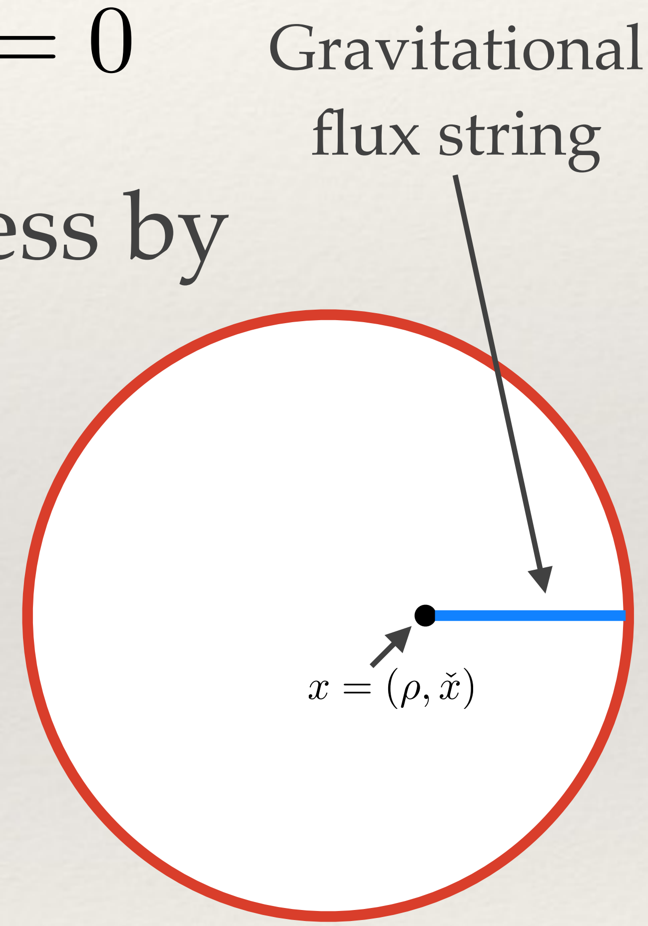
Globally: $\Phi(x) = \phi(x^\mu + V^\mu(x))$ where $\delta V^\mu(x) = \kappa\xi^\mu(x)$

Infinitesimally: $\Phi(x) = \phi(x) + V^\mu(x)\partial_\mu\phi(x) + \mathcal{O}(\kappa^2)$

[Donnelly and Giddings, '15]

Wilson Line Dressing

- Scalar field in a *particular* gauge is diffeomorphism invariant. E.g. Fefferman-Graham gauge $h_{\rho\mu} = 0$
- Suppose x^μ are non-FG coordinates. Then dress by linearized transformation back to FG gauge:



$$V^\rho(x) = \frac{\kappa\cos\rho}{2l^2}\int_\rho^{\pi/2} du\ h_{\rho\rho}(u,\check{x})\cos(u)$$

$$V^\tau(x) = -\frac{\kappa}{l^2}\left[\int_\rho^{\pi/2} ds\ \cos^2(u)\left(h_{\rho\tau}(u,\check{x}) + \frac{1}{2\cos(u)}\partial_\tau\int_u^{\pi/2} du'\ h_{\rho\rho}(u',\check{x})\cos(u')\right)\right]$$

$$V^\theta(x) = \frac{\kappa}{l^2}\left[\int_\rho^{\pi/2} ds\ \tan^2(u)\left(h_{\rho\theta}(u,\check{x}) + \frac{1}{2\cos(u)}\partial_\theta\int_u^{\pi/2} ds'\ h_{\rho\rho}(u',\check{x})\cos(u')\right)\right]$$

- The resulting dressed field is nonlocal, as required to satisfy the gravitational Gauss law

Bulk Commutators

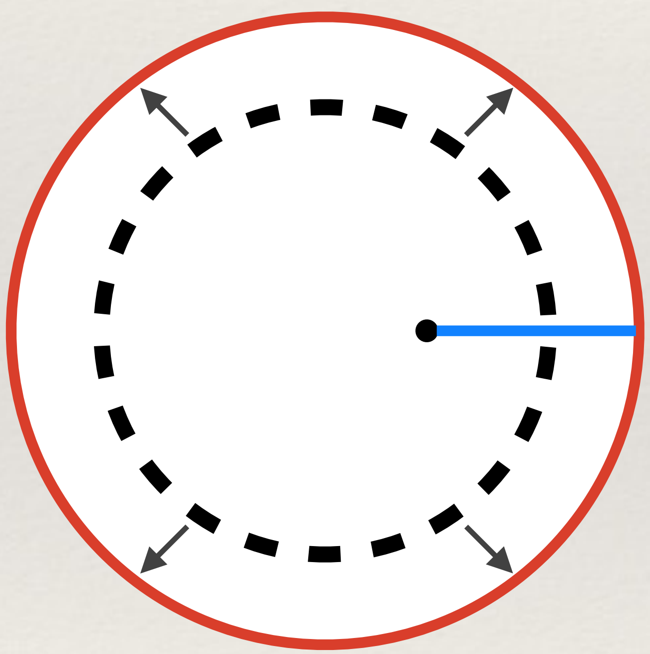
- With covariant gauge breaking, dressing creates algebra
- Example: Time evolution of dressed scalar.

$$[H,\Phi(x)] = \cancelto{0}{[H,\phi(x)]} + [H, V^{\mu}\partial_{\mu}\phi(x)] + \mathcal{O}(\kappa)$$

$$= [H, V^{\mu}]\,\partial_{\mu}\phi(x) + \mathcal{O}(\kappa)$$

$[H, V^{\mu}] = -i\delta^{\mu}_{\tau}$

$$= -i\partial_{\tau}\Phi(x) + \mathcal{O}(\kappa)$$



Surface for Hamiltonian goes to infinity, but always intersects dressing

- If we fix a gauge, then Dirac brackets ensure the correct commutators. E.g. Fefferman-Graham gauge:

$$\Phi_{\mathrm{FG}}(x) = \phi(x) \qquad\qquad [H,\phi(x)]_{\mathrm{Dirac}} = -i\partial_{\tau}\phi(x)$$

Boundary Commutators

- How does boundary algebra arise from bulk algebra? Can't be a naive limit, because:

Boundary limit of time translations disagree:

$$\lim_{\rho\to\pi/2}(\cos\rho)^{-\Delta}\,[H,\Phi(\rho,\check{x})] = -i\partial_\tau\mathcal{O}(\check{x})$$

$$\lim_{\rho\to\pi/2}(\cos\rho)^{-\Delta}\,[H,\phi(\rho,\check{x})] = 0$$

Extrapolated operators agree because dressing vanishes at infinity:

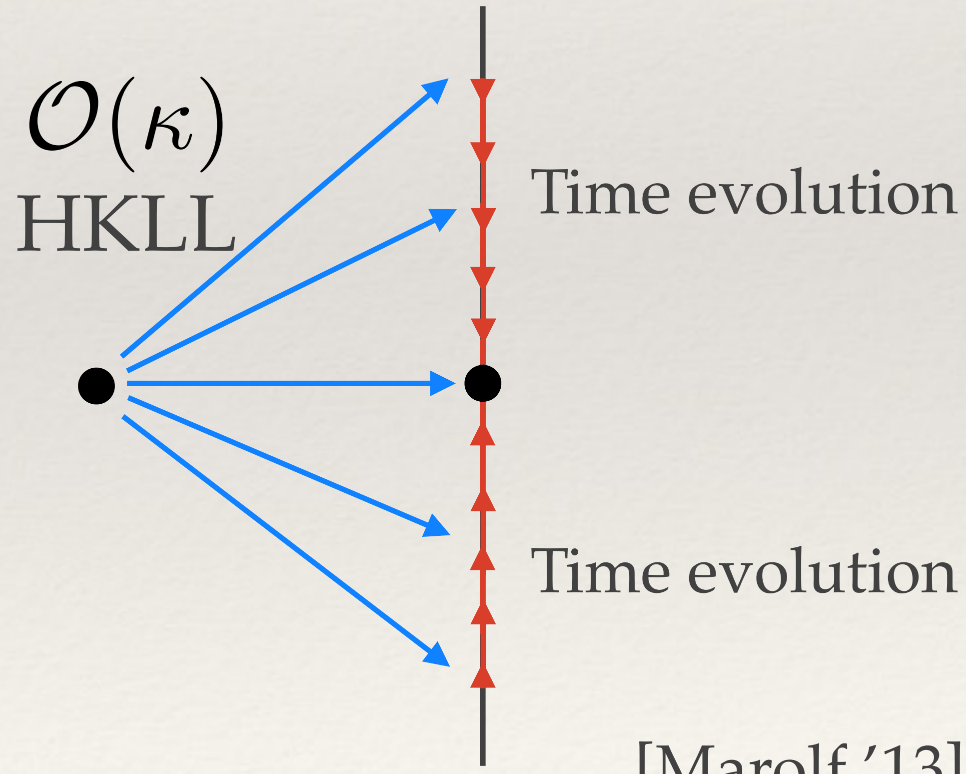
$$\lim_{\rho\to\pi/2}(\cos\rho)^{-\Delta}\Phi(\rho,\check{x}) = \mathcal{O}(\check{x})$$

$$\lim_{\rho\to\pi/2}(\cos\rho)^{-\Delta}\phi(\rho,\check{x}) = \mathcal{O}(\check{x})$$

- Some remnant of dressing must be preserved by extrapolate map

Ongoing Work: Relation to Holography

- Goal 1: Construct algebra of boundary observables
- Goal 2: Explicit understanding of equal-time reconstruction argument for holography
    - How to perform $\mathcal{O}(\kappa)$ HKLL for dressed operators?
    - How to understand acausality?



[Marolf '13]
[Heemskerk, Marolf, Polchinski, Sully '12]
[Hamilton, Kabat, Lifschytz, Lowe '06]